



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A



ALGORITHMS FOR MONLINEAR PROGRAMMING

FINAL REPORT 83-1

BY

DONALD GOLDFARB

JULY 1983

U. S. ARMY RESEARCH OFFICE

CONTRACT MUMBER DAAG 29-82-K-0163

THE CITY COLLEGE OF THE CITY UNIVERSITY

OF NEW YORK

DEPARTMENT OF COMPUTER SCIENCE

NEW YORK, N.Y. 10031

APPROVED FOR PUBLIC RELEASE:
DISTRIBUTION UNLIMITED



DIE FILE COPY

THE VIEW, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS REPORT ARE THOSE OF THE AUTHOR(S) AND SHOULD NOT BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION, POLICY, OR DECISION, UNLESS SO DESIGNATED BY OTHER DOCUMENTATION.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PA	GE READ INSTRUCTIONS BEFORE COMPLETING FORM
. REPORT NUMBER	GOVT ACCESSION NO. 3. RECIPIENT'S CATALOG NUMBER
83-1	D-A130 963
. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
•	Final report
Algorithms for Nonlinear Programming	1 August 1982 - 1 June 1983
	6. PERFORMING ORG. REPORT NUMBER
. AUTHOR(s)	B. CONTRACT OR GRANT NUMBER(*)
Donald Goldfarb	DAAG 29-82-K-0163
. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
The City College of the City Univers	ity
of New York, Dept. of Computer Scien	
New York, N.Y. 10031	
1. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
U. S. Army Research Office	July 1983
Post Office Box 12211	13. NUMBER OF PAGES
Research Triangle Park, NC 27709	4
14. MONITORING AGENCY NAME & ADDRESS(If different fr	om Controlling Office) 15. SECURITY CLASS. (of this report)
Department of the Navy	
Office of Naval Research Resident Re	presentative Unclassified
715 Broadway (5th floor)	Unclassified Ise DECLASSIFICATION/DOWNGRADING SCHEDULE
New York, N.Y. 10003	2CHEDULE
The state of the s	

16 DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Optimization, Linear Programming, Quadratic Programming, Nonlinear Programming, Computational Complexity, Relaxation Methods, Finite Difference Jacobians and Hessians, Jacobi, Gauss-Seidel, SOR, Shadow--Vertex Method, Assignment Problem, Dual Simplex Method.

20. ABSTRACT (Continue on reverse elde if necessary and identify by block number)

Several algorithms in linear, quadratic, and nonlinear programming have been developed and analyzed. These include:

- (i) The development of relaxation methods for finding a feasible solution to a system of linear inequalities based upon generating surrogate constraints.
- (ii) The worst-case behaviour of the shadow-vertex simplex algoritm was shown to be the exponential.

DD 1 JAN 73 1473 EDITION OF THOU 65 IS OBSOLETE

- (iii) Necessary and sufficient conditions for versions of iterative methods, including the Jacobi, Gauss-Seidel and SOR methods, designed for solving equality constrained quadratic programs have been obtained.
- (iv) The development of numerically stable and efficient implementations of primal methods for quadratic programming.
- (v) The development of optimal algorithms for estimating Jacobian and Hessian matrices arising in finite difference calculations.

Accession For	7
NTIS GRA&I	1
DTIC TAB	1
Unannounced	1
Justification	-
Ву]
Distribution/]
Availability Codes	
Avert and/or	1 1 m a)
Dist Dynamal	\ \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
1.1	
I W	
	i

A. PROBLEM STATEMENT

Our research under this grant has focused on the development and analysis of algorithms for mathematical programming. The algorithms that were developed and studied can be categorized according to the areas of linear, quadratic, and nonlinear programming.

In linear programming a number of problems were studied including
(i) relaxation methods for finding a feasible solution to a system of linear inequalities, (ii) the computational complexity of the "shadow-vertex" simplex algorithm, and (iii) methods for solving the assignment, or maximum weighted matching problem.

In quadratic programming we studied (i) iterative methods for solving equality constrained quadratic programs and (ii) how to implement primal methods in a numerically stable and efficient way.

In the area of nonlinear programming we considered the problem of estimating Hessian and Jacobian matrices with a minimal number of gradient and function differences in certain special cases.

B. SUMMARY OF RESEARCH RESULTS

1. Linear Programming Algorithms

- (i) Several relaxation methods have been developed for finding a feasible point for a system of linear inequalities. These methods are based upon generating "surrogate" linear inequalities, a technique developed in our earlier work on the ellipsoid method. Preliminary computational testing of these methods has been performed and some convergence results have been obtained. Further testing and theoretical analysis is planned.
- (ii) There is currently a great deal of interest in the computational complexity of algorithms, both in the sense of worst-case behavior and average-case behavior. Because of its extensive use, the computational complexity of the simplex method is of particular importance. It has been known for some time that some of the standard versions of the simplex method have exponential worst-case time bounds. (This was proved for the steepest edge simplex method by the principal investigator.) Recently, a variant of the simplex algorithm, the "shadow-vertex algorithm" has been shown to be polynomial in both the number of the variables and constraints on the average. We have shown that in the worst case this variant can still require an exponential number of pivots.
- (iii) Work has begun on the development of an efficient dual simplex algorithm for solving the assignment problem.

2. Quadratic Programming Algorithms

(i) We have obtained necessary and sufficient conditions for the convergence of a class of iterative methods for solving equality constrained quadratic programs. These methods include generalizations of the Jacobi, Gauss-Seidel and SOR methods. Our current goal is to study how to use these methods within successive quadratic programming algorithms for solving large nonlinear programming problems.

2. (Cont...)

(ii) Our work on the development of efficient and numerically stable methods for solving quadratic programming problems has continued. Based on our earlier work with Dr. A. Idnani on dual and primal-dual quadratic programming methods we have developed efficient and numerically stable implementations for purely primal methods.

3. Nonlinear Programming Algorithms

(i) Methods for estimating Jacobian and Hessian matrices using a minimal number of function and gradient differences have been developed for certain special cases. The cases studied were finite difference approximations to partial differential equations. The optimal schemes that were developed were based upon the "computational molecule" or "stencil" of the underlying finite difference operator. A "tearing" approach which is applicable to more complicated structures was also developed.

C. PUBLICATIONS

- D. Goldfarb and Ph.L. Toint "Optimal Estimation of Jacobian and Hessian Matrices that Arise in Finite Difference Calculations", submitted to Mathematics of Computation.
- D. Goldfarb "Worst-Case Complexity of the Shadow Vertex Simplex Algorithm" Columbia University IE&OR Technical Report (to be submitted to a journal)

D. PARTICIPANTING SCIENTIFIC PERSONNEL

Principal Investigator: Professor Donald Goldfarb

